AD-A095 041 RAND CORP SANTA MONICA CA F/G 12/1
ESTIMATING THE AUTOCORRELATED ERROR MODEL WITH TRENDED DATA: FU--ETC(U)
NOV 79 R E PARK, B M MITCHELL
UNCLASSIFIED RAND/R-2470 NL

1 OF 1
AD
A0 95041

END
DATE
FILMED
3-81
DTIC

ESTIMATING THE AUTOCORRELATED ERROR MODEL WITH TRENDED DATA: FURTHER RESULTS

ROLLA EDWARD PARK, BRIDGER M. MITCHELL

R-2470
NOVEMBER 1979

Rand
SANTA MONICA, CA. 90406

ISBN 0-8330-0193-0
LC Card No. 79-92662

The Rand Publications Series: The Report is the principal publication documenting and transmitting Rand's major research findings and final research results. The Rand Note reports other outputs of sponsored research for general distribution. Publications of The Rand Corporation do not necessarily reflect the opinions or policies of the sponsors of Rand research.

Published by The Rand Corporation

ESTIMATING THE AUTOCORRELATED ERROR MODEL WITH TRENDED DATA: FURTHER RESULTS

ROLLA EDWARD PARK, BRIDGER M. MITCHELL

R-2470
NOVEMBER 1979

Rand
SANTA MONICA, CA. 90406

PREFACE

Beginning with Rao and Griliches (1969), several studies have compared the efficiency of various estimators of regression models with first-order autocorrelated errors. The present report, however, is the first to compare the efficiency of *all principal estimators* with *trended data* and also to compare their performance in *hypothesis testing*.

This report is an outgrowth of work undertaken to guide the analysis of school district expenditure behavior, but it should be of interest to applied econometricians working in many different substantive fields. Earlier results are reported in Park and Mitchell (1978). The present report adds iterative methods to the list of estimators compared before, and provides more extensive and illuminating measures of relative performance. The Rand Corporation provided computer time and secretarial services to make this new report possible.

This study is also forthcoming in the *Journal of Econometrics*.

on file Per Ltr

A

ABSTRACT

A Monte Carlo study is made of the small sample properties of various estimators of the linear regression model with first-order autocorrelated errors. When independent variables are trended, estimators using T transformed observations (Prais-Winsten) are much more efficient than those using T-1 (Cochrane-Orcutt). The best of the feasible estimators is iterated Prais-Winsten using a sum-of-squared-error minimizing estimate of the autocorrelation coefficient ρ. None of the feasible estimators performs well in hypothesis testing; all seriously underestimate standard errors, making estimated coefficients appear to be much more significant than they actually are.

ACKNOWLEDGMENTS

We are grateful to Professor James MacKinnon, Queen's University, and the *Journal of Econometrics* referees for many helpful suggestions that have substantially improved this report. Remaining shortcomings are definitely our own fault.

CONTENTS

I. INTRODUCTION AND SUMMARY

Several estimators are commonly used to estimate the linear regression model with first-order autocorrelated errors. This Monte Carlo study extends the investigation of the small-sample properties of such estimators, first undertaken by Rao and Griliches (1969), in two major respects: (1) We provide a systematic comparison of the estimation efficiency of *all principal estimators* with *trended data* and (2) we compare estimator performance in *hypothesis testing*.[1]

The major estimators can be classified according to (a) whether T-1 or T transformed observations are used, (b) whether the autocorrelation coefficient ρ is known or estimated, and, if estimated, (c) whether the estimate of ρ is iterated. With trended data we find that estimators using T-1 observations have very low efficiency, often less than that of ordinary least squares (OLS), regardless of whether ρ is known or estimated. For unknown ρ, iterative estimators using all T observations dominate OLS and are somewhat more efficient than two-step estimators. We find that the iterated Prais-Winsten estimator using the sum-of-squares minimizing ρ estimate performs marginally better than the full maximum likelihood estimator.

For the empirical researcher, reliable hypothesis testing procedures are as important as efficient coefficient estimates. Perhaps the most serious deficiency of OLS in the presence of autocorrelation is not inefficiency but bias in its estimated standard errors--a bias that in many situations will make the estimated coefficients appear to be much more significant than they actually are. Unfortunately, our results show that in this regard the preferred estimators, though substantially better than OLS, can still be seriously misleading.

---

[1] There has been a good deal of recent work on particular aspects of estimating the autocorrelated error model, but none treats all of the principal estimators and none deals with hypothesis testing. See the discussion below (p. 8).

The estimation problem and the estimators that we consider are described in the next section. For short time series (T=20), Section III compares the efficiency of the various estimators and Section IV describes their performance in hypothesis testing. Results for longer time series (T=50) are presented in Section V. Section VI is a concise list of recommendations based on our results.

II. THE ESTIMATION PROBLEM

THE MODEL

The commonly encountered econometric model is:

$$y_t = x_t \beta + u_t \, ,$$

$$u_t = \rho u_{t-1} + \varepsilon_t \, , \; t = 1, 2, \ldots, T; \tag{1}$$

where $|\rho| < 1$, $E(\varepsilon) = 0$, $E(\varepsilon\varepsilon') = \sigma_\varepsilon^2 I$. In general, x_t will include a 1 for the constant term; that is,

$$x_t = [1, x_{2,t}, \ldots, x_{K,t}] \, . \tag{2}$$

For this model the T x T covariance matrix of the error vector is

$$E(uu') = \sigma_u^2 V = \sigma_u^2 \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{T-3} \\ \vdots & \vdots & \vdots & & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{bmatrix}, \tag{3}$$

where $\sigma_u^2 = \sigma_\varepsilon^2/(1-\rho^2)$.

If ρ is known, the AITKEN estimator

$$b = (X'V^{-1}X)^{-1} X'V^{-1}y \tag{4}$$

is best linear unbiased. Computationally, it is convenient to decompose $V^{-1} = [1/(1-\rho^2)]R'R$,

where

$$R = \begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & \cdots & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \tag{5}$$

and calculate

$$b = (X'R'RX)^{-1} X'R'Ry \tag{6}$$

as an OLS regression of the transformed variables $y^* = Ry$ on $X^* = RX$.

THE ESTIMATORS

The estimators that we consider may all be thought of as variants on the AITKEN estimator. As shown in Fig. 1, some use T transformed observations, and some use T-1; they also use different values for ρ.

Estimated Autocorrelation Coefficient ($\hat{\rho}$)	Number of Observations: T-1	Number of Observations: T
Zero	--	OLS
True ρ	TRUECO	AITKEN
Sum-of-squared-error minimizing	2SCO ITERCO	2SPW ITERPW
Likelihood maximizing	--	BM

Fig. 1--Estimators considered in this report.

It is common to omit the first row in the transformation matrix R.[1] We denote the reduced matrix by S. Then the transformed variables $y^* = Sy$ and $X^* = SX$ are the T-1 weighted first differences

$$y_t^* = y_t - \rho\, y_{t-1}$$

$$x_t^* = [1-\rho,\ x_{2,t}-\rho x_{2,t-1},\ \ldots,\ x_{K,t}-\rho x_{K,t-1}] \ . \tag{7}$$

This is the transformation first proposed by Cochrane and Orcutt (CO, 1949).

Alternatively, Prais and Winsten (PW, 1954) recommend retaining the first row of R, in which case one has T transformed observations, including in addition to (7) the transformed first observation

$$y_1^* = \sqrt{1-\rho^2}\, y_1$$

$$x_1^* = [\sqrt{1-\rho^2},\ \sqrt{1-\rho^2}\, x_{2,1},\ \ldots,\ \sqrt{1-\rho^2}\, x_{K,1}] \ . \tag{8}$$

If the true value of ρ were known, its use in R with all T observations would yield the AITKEN estimator. Using true ρ and T-1 observations would give what we call the TRUECO (true ρ, Cochrane-Orcutt) estimator.

In practice, ρ is almost never known. It is common to substitute a consistent estimate $\hat{\rho}$ based on the residuals $\hat{u}_t$ from a first-stage OLS regression using untransformed variables. The estimators we use minimize the sum of squared errors for the transformed regression, conditional on given estimates of β.[2] For the CO transformation, the estimator is

---

[1] This is done, for example, in the widely used regression package TSP; the TSP procedure CORC is the same as what we call ITERCO.

[2] We are grateful to James MacKinnon for suggesting that we use the sum-of-squared-errors minimizing estimate of ρ. See below (p. 6) for a discussion of alternative estimators.

$$\hat{\rho}_{CO} = \sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1} / \sum_{t=1}^{T-1} \hat{u}_t^2 \tag{9a}$$

and for the PW transformation it is

$$\hat{\rho}_{PW} = \sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1} / \sum_{t=2}^{T-1} \hat{u}_t^2 . \tag{9b}$$

Using $\hat{\rho}_{CO}$ in S produces what we call 2SCO (two-stage Cochrane-Orcutt) estimates; using $\hat{\rho}_{PW}$ in R produces 2SPW (two-stage Prais-Winsten) estimates.

Iterative estimates based on estimated ρ are obtained as follows: (a) Use the second stage estimate of β to calculate new residuals $\hat{u} = y - Xb$. (b) Use these to calculate a new estimate of ρ. (c) Use the new $\hat{\rho}$ in S (or R) to calculate a new estimate of β. (d) Repeat these steps until successive estimates of ρ differ by less than $\pm\delta$. We set $\delta = .00001$ and call the resulting estimators ITERCO (iterated Cochrane-Orcutt) using S and ITERPW (iterated Prais-Winsten) using R.

There is some chance that $\hat{\rho}$ estimated according to (9a) or (9b) will take on inadmissible values ($|\hat{\rho}| \geq 1$). When $\hat{\rho} \geq 1$, we reset it to .99999 ($= 1 - \delta$); $\hat{\rho} \leq -1$ becomes $-.99999$.

Finally, Beach and MacKinnon (BM, 1978) proposed a full maximum likelihood estimator. Because the log likelihood function includes the term $.5\log(1 - \rho^2)$, the estimated ρ is bounded away from ± 1, so that inadmissible values of $\hat{\rho}$ do not occur. Computationally, the BM procedure is the same as ITERPW, except that a different estimate of ρ is used; the BM estimate of ρ maximizes the likelihood function conditional on estimated β.

ALTERNATIVE ESTIMATORS OF ρ

Many previous Monte Carlo studies have used the following estimate of ρ:

$$\hat{\rho} = \sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1} / \sum_{t=2}^{T} \hat{u}_t^2 . \tag{10}$$

This estimator is consistent, but unlike (9a) or (9b) it does not minimize the sum-of-squared-errors for either CO or PW.

In preparing this report we found (10) to be inferior. Using (9b) rather than (10) in 2SPW and ITERPW reduces the well-known downward bias in estimated ρ, and results in slightly smaller root mean squared errors for both $\hat{\rho}$ and b in almost all cases. Using (9a) rather than (10) makes little difference in 2SCO, but has a large effect in ITERCO on the number of times $\hat{\rho}$ sticks at the boundary value .99999. With (10), $\hat{\rho}$ = .99999 in a large fraction of the experiments in which true $\rho \geq .8$--over 50 percent in one case. With (9a), the fraction never exceeded 4 percent, and it was usually much smaller than that.[1] Since boundary estimates of ρ result in very bad estimates of the intercept coefficient β_1 (for reasons discussed in Section III below), (9a) is decidedly preferable to (10) in ITERCO.

Other consistent estimators of ρ have been proposed. Theil (1971, p. 254) suggests incorporating a degrees-of-freedom correction that would yield estimates smaller in absolute value than (9a) and (9b). In light of the downward bias in (9a) and (9b) as they stand, this correction does not seem desirable.

Durbin (1960) proposes running an auxiliary regression to estimate ρ. Using (9a) in ITERCO and (9b) in either 2SPW or ITERPW almost always resulted in ρ estimates that were less biased and had smaller mean squared errors than the Durbin estimates.[2]

On balance, the sum-of-squared-errors minimizing ρ estimators (9a) and (9b) appear to be better than any of the commonly used alternatives.

---

[1]See Appendix Table A.8. The reason for the difference is that $\hat{u}_1^2$ tends to be larger than $\hat{u}_T^2$ in ITERCO, because of the relatively small weight given the first observation in the CO transformed regression.

[2]Compare Appendix Table A.6 with Table 2 in Park and Mitchell (1978, p. 12).

INDEPENDENT VARIABLES

We analyze the relative performance of the estimators in Fig. 1 using three independent variables.[1] They are one artificial trended series:

(a) $x_t = [1, t]$;

one real trended series:

(b) $x_t = [1, GNP_t]$, the annual U.S. gross national product in constant dollars beginning in 1950;

and, for comparison, one real untrended series:

(c) $x_t = [1, CAP_t]$, the annual U.S. manufacturing capacity utilization rate, also beginning in 1950.

We work chiefly with 20 observations (T=20), a sample size representative of many time series studies. In Section V we discuss the differences found when 50 (quarterly) observations are available.

Strictly speaking, our results are all conditional on the particular X matrices we have used. But we believe that our findings are generally applicable for trended independent variables, because the results are very much the same for the artificial and the real trended series. Each type of series answers questions left open by the other. The artificial series clearly establishes the effect of pure trends, but leaves open the question of whether the results would hold for the quirks present in real-world data--a question that the GNP results answer in the affirmative.[2]

OTHER RECENT WORK

Although several recent papers have discussed aspects of estimating the autocorrelated error model, this is the first to provide a unified investigation of all of the major estimators with trended data. Furthermore, with the exception of Park and Mitchell (1978), none of the previous work has taken up the question of hypothesis testing.

---

[1]Maeshiro (1976) also used (a) and (b). Instead of (c), he used quarterly capacity utilization starting with 1948.

[2]Furthermore, Monte Carlo experiments with quite different artificial time series yielded similar results; see Park and Mitchell (1979).

In a pair of articles, Maeshiro compares the efficiency of estimators using *known* ρ. In (1976), he shows that TRUECO is less efficient than OLS with trended data, and in (1979) he demonstrates that AITKEN is often substantially more efficient than TRUECO. Beach and MacKinnon (1978) compare their proposed full maximum likelihood estimator, BM, with ITERCO, and find BM to be more efficient, especially when the data are trended. Harvey and McAvinchey, in an unpublished paper (1978), make efficiency comparisons of most of the major estimators applied to both trended and untrended data. They do not, however, consider ITERPW, the estimator that we find to be the best performer. Park and Mitchell (1978) do not consider any iterative procedures. Using untrended data, Spitzer (1979) revisits the estimators investigated by Rao and Griliches (1969)--these do not include ITERCO and ITERPW--and adds BM.

III. EFFICIENCIES OF ESTIMATORS

EXACT THEORETICAL EFFICIENCIES

We can make two of our efficiency comparisons using exact formulas. For the case of known values of ρ, the exact variances of the OLS, TRUECO, and AITKEN estimators are given by the formulas:

$$\mathrm{var}(b_{\mathrm{OLS}}) = \sigma_u^2 (X'X)^{-1} X'VX(X'X)^{-1} \tag{11}$$

$$\mathrm{var}(b_{\mathrm{TRUECO}}) = \sigma_\varepsilon^2 (X'S'SX)^{-1} \tag{12}$$

$$\mathrm{var}(b_{\mathrm{AITKEN}}) = \sigma_u^2 (X'V^{-1}X)^{-1} = \sigma_\varepsilon^2 (X'R'RX)^{-1} \tag{13}$$

For these estimators we define relative efficiency as the *square root* of the ratio of the variances of the estimators being compared. For example:

$$\mathrm{EFF}(b_{1,\mathrm{CO}}) = [\mathrm{var}(b_{1,\mathrm{OLS}})/\mathrm{var}(b_{1,\mathrm{TRUECO}})]^{.5} \quad .$$

This definition is in accord with comparisons of standard errors or t-ratios commonly used by applied researchers; to use the ratio itself would make the difference between estimators appear larger than they "really" are.

EXPERIMENTAL EFFICIENCIES

We used Monte Carlo simulation to assess the relative efficiencies for the other five estimators--2SCO, ITERCO, 2SPW, ITERPW, and BM. For each of the independent variables x and for each value of $\rho = -.8, .0, .4, .8, .9, .98$, we generated 1000 samples using model (1) with $\beta = [1, 1]$. A value u_0 was generated by drawing a random ε_0 from N(0, 1) and dividing by $\sqrt{1-\rho^2}$. Successive values

of ε_t drawn from N(0, 1) were used to calculate $u_t = \rho u_{t-1} + \varepsilon_t$, and hence $y_t = x_t\beta + u_t$. We then applied each estimation method and averaged the squared errors of the estimated coefficients over the 1000 samples.[1] For these estimators we define relative efficiency as the ratio of the root mean squared errors of the estimators being compared. For example:

$$\mathrm{EFF}(b_{1,2SCO}) = \mathrm{RMSE}(b_{1,OLS})/\mathrm{RMSE}(b_{1,2SCO}) ,$$

where

$$\mathrm{RMSE}(b_1) = [\Sigma_1^{1000}(b_1-\beta_1)^2/1000]^{.5} .$$

EFFICIENCY OF ESTIMATORS THAT USE T-1 TRANSFORMED OBSERVATIONS

Table 1 shows the efficiency, relative to OLS, of the three estimators that use T-1 observations. Here we focus on the results for positive ρ.[2] For trended variables all three estimators are less efficient than OLS in almost all of the cases tabulated.

For x_t = [1, t], TRUECO has extremely low efficiency as ρ approaches 1. This poor performance is the result of collinearity; because the transformed variable $x^*_{2,t} = x_{2,t}-\rho x_{2,t-1}$ approaches the same value for all t, the T-1 vector x^*_2 becomes collinear with the transformed vector for the constant term $x^*_{1,t} = 1-\rho$, a situation well-known for producing inefficient estimates. More generally, the CO transformation of any linearly trended variable (using $\rho > 0$) produces observations that are more nearly constant than are the

[1]The calculations were done in double precision on an IBM 370-158 using regression analysis subroutines from the STATLIB statistical package.

[2]Results for $\rho = -.8$ and $\rho = .0$ are included in the Appendix tables.

Table 1

EFFICIENCY, RELATIVE TO OLS, OF ESTIMATORS
THAT USE T-1 TRANSFORMED OBSERVATIONS
(T=20)

Independent Variable	Estimator	ρ = .4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
t	TRUECO	.81	.86	.50	.62	.29	.42	.04	.11
	2SCO	.81	.86	.64	.77	.31	.62	.66	.74
	ITERCO	.80	.85	.51	.69	.27	.56	.54	.64
GNP_t	TRUECO	.88	.91	.71	.81	.57	.75	.29	.71
	2SCO	.91	.93	.84	.91	.87	.95	.95	1.03
	ITERCO	.73	.85	.59	.80	.51	.83	.52	.88
CAP_t	TRUECO	1.10	1.10	1.85	1.83	2.10	2.19	1.04	2.51
	2SCO	1.05	1.04	.01	1.41	.00	1.65	.00	1.83
	ITERCO	1.03	1.03	.01	1.65	.00	2.03	.00	2.27

Note: Exact theoretical relative efficiency for TRUECO; experimental relative efficiency for 2SCO and ITERCO.

untransformed values, and hence more nearly collinear with the constant vector.[1]

When ρ is large, 2SCO using trended data performs better than TRUECO.[2] However, it is less efficient than OLS in almost all cases. Iterating on $\hat{\rho}$ makes matters worse. For trended variables, ITERCO is less efficient than 2SCO in all of the cases tabulated.

For the untrended variable CAP the picture is mixed. All three CO slope estimators are more efficient than OLS, but 2SCO and ITERCO produce very bad intercept estimates when $\hat{\rho}$ takes on the boundary value .99999. This happened between 3 and 35 times out of 1000 trials in the cases tabulated,[3] causing very large root mean squared errors and resulting in near-zero relative efficiency.

Because a few very bad estimates can dominate the experimental relative efficiences, Table 1 might conceivably hide a good performance in most of the trials. This is *not* the case for trended variables, although it is true for CAP. Table 2 shows the number of times out of 1000 that each estimator came closer to the true value of the coefficient than did OLS. For trended variables, none of the CO estimators was closer than OLS as much as half of the time in any case tabulated.[4]

---

[1]Maeshiro (1976) explains the poor performance of the TRUECO estimator as a result of reduction of variance of the transformed independent variables. In one sense this is equivalent to our "collinearity" explanation, because a low-variance independent variable is represented by a nearly constant vector, which is nearly collinear with the vector for the constant term. But in another sense it is somewhat misleading. In a model without a constant term, an independent variable with low (or even zero) variance causes no problems. For example, the estimator of β in $y_t = \beta x_t + u_t$, $E(u_t) = 0$, when $x_t = k$ for all t has variance $\mathrm{var}(b) = \sigma^2/Tk^2$.

[2]This refutes Maeshiro's (1976) conjecture that "an estimator utilizing relevant extraneous information (e.g., the true value of ρ) cannot be less efficient than an estimator not utilizing it." The reason, paradoxically, is the downward bias in $\hat{\rho}$ (see Appendix Table A.7). The higher the value of ρ used in the transformation, the more collinear are the transformed variables.

[3]See Appendix Table A.8.

[4]The proportion is significantly less than one-half on a binominal test at the .05 significance level in all but two cases.

Table 2

NUMBER OF TIMES IN 1000 TRIALS THAT ESTIMATORS THAT
USE T-1 TRANSFORMED OBSERVATIONS BEAT OLS
(T=20)

Independent Variable	Estimator	ρ							
		.4		.8		.9		.98	
		b_1	b_2	b_1	b_2	b_1	b_2	b_1	b_2
t	TRUECO	371	394	236	316	151	222	24	74
	2SCO	382	413	325	378	324	349	352	397
	ITERCO	381	410	321	372	318	340	347	383
GNP_t	TRUECO	443	461	382	441	324	398	192	374
	2SCO	443	457	421	468	433	454	438	495
	ITERCO	441	453	405	451	395	423	395	450
CAP_t	TRUECO	568	563	726	731	728	713	523	781
	2SCO	547	534	737	737	742	741	721	793
	ITERCO	532	521	722	725	720	732	703	786

Note: Counts greater than 531 or smaller than 469 are significantly different from 500 at the .05 level.

The lesson is clear: Avoid using any form of the Cochrane-Orcutt estimator.

EFFICIENCY OF ESTIMATORS THAT USE T TRANSFORMED OBSERVATIONS

Turning now to estimators that use all T transformed observations, we see in Table 3 that they all provide more efficient estimates than does OLS. By retaining the differentially weighted first observation, these estimators break the collinearity that plagued the CO estimators.

For trended data, the AITKEN estimator provides respectable, although not spectacular, efficiency improvements over OLS, ranging up to 26 percent in the cases tabulated. The efficiency gain is highest in just those cases where CO performs worst, that is, for $\rho \geq .8$. The other methods, using estimated rather than true ρ, preserve about half of the AITKEN efficiency improvement. For untrended data, the efficiency gains are larger, and more of the gain is retained when ρ must be estimated.

Iteration helps. ITERPW is slightly more efficient than 2SPW using trended data, and substantially more efficient with untrended data. The BM estimator has virtually the same efficiency as ITERPW.

Using ITERPW as a standard of comparison, we show in Table 4 the number of times in 1000 trials that the other estimators using T transformed observations come closer to the true coefficient value. In almost all cases, ITERPW outperformed the other estimators that use estimated ρ.

Table 3

EFFICIENCY, RELATIVE TO OLS, OF ESTIMATORS THAT USE T TRANSFORMED OBSERVATIONS (T=20)

Independent Variable	Estimator	ρ = .4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
t	AITKEN	1.02	1.02	1.06	1.09	1.08	1.10	1.03	1.08
	2SPW	1.01	1.01	1.05	1.06	1.05	1.08	1.02	1.05
	ITERPW	1.01	1.01	1.06	1.07	1.05	1.08	1.02	1.05
	BM	1.01	1.01	1.05	1.06	1.05	1.07	1.02	1.04
GNP_t	AITKEN	1.02	1.02	1.13	1.14	1.16	1.20	1.13	1.26
	2SPW	1.01	1.01	1.08	1.09	1.10	1.13	1.06	1.12
	ITERPW	1.00	1.01	1.08	1.09	1.11	1.14	1.07	1.14
	BM	1.01	1.01	1.08	1.09	1.10	1.13	1.06	1.12
CAP_t	AITKEN	1.14	1.14	1.85	1.86	2.15	2.21	1.95	2.52
	2SPW	1.06	1.06	1.38	1.38	1.57	1.60	1.54	1.75
	ITERPW	1.04	1.05	1.61	1.61	1.94	2.00	1.77	2.15
	BM	1.05	1.06	1.58	1.58	1.92	1.97	1.75	2.12

Note: Exact theoretical relative efficiency for AITKEN; experimental relative efficiency for 2SPW, ITERPW, and BM.

Table 4

NUMBER OF TIMES IN 1000 TRIALS THAT ESTIMATORS THAT USE T TRANSFORMED OBSERVATIONS BEAT ITERPW (T=20)

Independent Variable	Estimator	ρ = .4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
t	AITKEN	517	503	534	537	517	546	563	542
	2SPW	493	514	449	445	430	420	458	439
	BM	489	510	446	445	450	406	433	432
GNP_t	AITKEN	532	532	527	527	564	591	568	550
	2SPW	502	499	433	428	391	383	419	377
	BM	499	494	438	433	382	380	409	364
CAP_t	AITKEN	561	565	561	564	523	531	539	538
	2SPW	500	500	398	390	399	423	434	417
	BM	512	514	430	419	436	449	427	423

Note: Counts greater than 531 or smaller than 469 are significantly different from 500 at the .05 level.

IV. HYPOTHESIS TESTING

In our opinion, the most troublesome characteristic of OLS when $V \neq I$ is not the loss in efficiency, but the bias in the statistic $\hat{\sigma}_u^2(X'X)^{-1}$, which is conventionally used as an estimator of the covariance matrix of the estimated coefficients. How serious this bias can be is illustrated by the experimental results shown in Table 5. We focus on the methods that use all T transformed observations, since they are always more efficient than methods using T-1 observations. OLS seriously underestimates standard errors for all cases shown. For example, for trended data and $\rho \geq .8$, when one applies a two-tailed test at the .05 level, the underestimate is large enough to lead one to judge an estimated coefficient to be significantly different from its true value 45 to 85 percent of the time.

The AITKEN estimator provides unbiased variance estimates, of course. Unfortunately, the procedures using estimated ρ do not do as well, although they do improve on OLS.[1] ITERPW is the best of the lot. Still, for $\rho \geq .8$ and trended data, it would result in rejection of a correct null hypothesis at least 25 percent of the time.

For the untrended variable CAP, the results are qualitatively similar, but the biases are less severe than in the case of trended variables.

On hypothesis testing grounds, ITERPW appears to be superior to both 2SPW and BM.[2]

---

[1]The covariance matrix of the estimated coefficients is estimated directly in the transformed regressions as $\hat{\sigma}_\varepsilon^2(X^{*\prime}X^*)^{-1}$, where $\hat{\sigma}_\varepsilon^2 = \hat{\varepsilon}'\hat{\varepsilon}/(T-K)$.

[2]Had we used the maximum likelihood estimate $\tilde{\sigma}_\varepsilon^2 = \hat{\varepsilon}'\hat{\varepsilon}/T$ in the BM procedure, the margin of superiority of ITERPW over BM would have been slightly larger.

Table 5

NUMBER OF TYPE 1 ERRORS IN 1000 TRIALS
AT .05 SIGNIFICANCE LEVEL
(T=20)

Independent Variable	Estimator	ρ							
		.4		.8		.9		.98	
		b_1	b_2	b_1	b_2	b_1	b_2	b_1	b_2
t	OLS	193	197	502	490	645	571	848	709
	2SPW	125	132	302	293	411	340	690	473
	ITERPW	124	131	293	285	401	336	700	474
	BM	126	133	312	305	433	360	731	503
GNP_t	OLS	186	185	457	449	601	596	730	666
	2SPW	138	138	258	251	354	343	509	413
	ITERPW	136	136	254	246	343	322	486	397
	BM	139	138	261	258	375	352	534	432
CAP_t	OLS	147	143	304	294	341	323	407	322
	2SPW	110	106	153	149	154	144	211	137
	ITERPW	115	113	102	101	90	86	144	86
	BM	112	109	107	107	98	92	176	86

<u>Note:</u> Counts greater than 63 or smaller than 37 are significantly different from 50 at the .05 level.

V. LONGER TIME SERIES

In this section we investigate how the results change when 50, rather than 20, observations are available for one of our independent variables, $x_t = [1, GNP_t]$. In order to increase the length of the GNP time series to get 50 observations, we must shift from annual to quarterly observations. Because the quarterly series exhibits short-term fluctuations that are averaged out in the annual series, our T=50 series is less trended than the annual GNP series used above. It is, however, typical of longer economic time series.

A larger sample markedly improves the estimators of ρ. For example, when true $\rho = .9$, the mean value of $\hat{\rho}_{ITERPW}$ increased from .59 for T=20 to .80 for T=50.[1] The bias, although still clearly apparent, is greatly reduced.

Tables 6, 7, and 8 repeat the information in Tables 1 through 5 for T=50. For the most part, the conclusions for T=20 also apply for the larger sample size. Estimators using T-1 transformed observations are usually less efficient than OLS. Those using T observations always improve on OLS, and the margin is wider for the larger sample size. Also, methods using estimated ρ retain more of the AITKEN estimator's margin of improvement (reflecting the improved estimate of ρ). ITERPW appears to be slightly better than either 2SPW or BM.

Increased sample size does nothing to reduce the bias in the OLS estimated standard errors, but it does improve the ITERPW estimates. Nevertheless, ITERPW would still lead to rejection of a correct null hypothesis up to 30 percent of the time in the cases tabulated.

---

[1] See Appendix Table A.6.

Table 6

EFFICIENCY COMPARISONS FOR ESTIMATORS THAT
USE T-1 TRANSFORMED OBSERVATIONS
(T=50)

Independent Variable	Estimator	ρ = .4 b_1	ρ = .4 b_2	ρ = .8 b_1	ρ = .8 b_2	ρ = .9 b_1	ρ = .9 b_2	ρ = .98 b_1	ρ = .98 b_2
		Efficiency relative to OLS[a]							
GNP_t	TRUECO	.90	.91	.84	.87	.87	.94	.92	1.39
	2SCO	.91	.92	.85	.88	.91	.95	.02	1.13
	ITERCO	.91	.92	.70	.77	.72	.84	.02	1.12
		Number of times in 1000 trials that other estimators beat OLS[b]							
GNP_t	TRUECO	427	428	406	430	433	461	445	599
	2SCO	425	432	401	416	449	477	509	553
	ITERCO	425	430	387	400	399	429	423	512

[a]Exact theoretical relative efficiency for TRUECO; experimental relative efficiency for 2SCO and ITERCO.

[b]Counts greater than 531 or smaller than 469 are significantly different from 500 at the .05 level.

Table 7

EFFICIENCY COMPARISONS FOR ESTIMATORS THAT
USE T TRANSFORMED OBSERVATIONS
(T=50)

Independent Variable	Estimator	ρ = .4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
		Efficiency relative to OLS[a]							
GNP_t	AITKEN	1.02	1.02	1.19	1.19	1.40	1.41	1.78	1.89
	2SPW	1.02	1.02	1.15	1.14	1.28	1.28	1.41	1.44
	ITERPW	1.02	1.02	1.12	1.11	1.26	1.26	1.52	1.57
	BM	1.02	1.02	1.14	1.13	1.28	1.29	1.48	1.52
		Number of time in 1000 trials that other estimators beat ITERPW[b]							
GNP_t	AITKEN	490	494	564	556	553	561	586	597
	2SPW	500	491	440	432	426	415	383	354
	BM	493	484	441	445	427	415	384	353

[a]Exact theoretical relative efficiency for AITKEN; experimental relative efficiency for 2SPW, ITERPW, and BM.

[b]Counts greater than 531 or smaller than 469 are significantly different from 500 at the .05 level.

Table 8

NUMBER OF TYPE 1 ERRORS IN 1000 TRIALS AT
.05 SIGNIFICANCE LEVEL
(T=50)

Independent Variable	Estimator	ρ = .4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
GNP_t	OLS	209	209	501	505	636	631	754	781
	2SPW	90	92	143	143	198	202	377	366
	ITERPW	87	92	138	137	184	191	307	296
	BM	90	92	151	151	200	208	357	338

Note: Counts greater than 63 or smaller than 37 are significantly different from 50 at the .05 level.

VI. RECOMMENDATIONS

Our results lead us to offer the following guidelines to practicing econometricians working with trended data in the presence of autocorrelation:

1. Avoid the Cochrane-Orcutt estimator (using T-1 transformed observations); it is more complicated than OLS and often less efficient.

2. Use the iterative version of the Prais-Winsten estimator (using T transformed observations). It offers efficiency gains over OLS that range from modest to substantial. It is slightly but clearly superior to two-stage Prais-Winsten. For trended data and a large autocorrelation coefficient, it also appears to have a slight edge, in small samples, over the full maximum likelihood method proposed by Beach and MacKinnon.

3. Distrust the conventional t-statistics. The OLS standard errors are vastly underestimated. The iterative Prais-Winsten standard errors are a substantial improvement, but still highly misleading. Because estimated coefficients seem much more significant than they really are, apply a more stringent confidence level for hypothesis testing.

APPENDIX TABLES

The tables in the text, above, show results for selected estimators for positive values of ρ only. The following Appendix tables include results for all relevant estimators for ρ = −.8, .0, .4, .8, .9, and .98.

Table	Title
A.1	Exact theoretical efficiency, relative to OLS, of estimators that use true ρ
A.2	Experimental efficiency of various estimators relative to OLS
A.3	Number of times in 1000 trials that various estimators beat OLS
A.4	Number of times in 1000 trials that various estimators beat ITERPW
A.5	Number of type 1 errors in 1000 trials at .05 significance level
A.6	Performance of estimators of ρ
A.7	Number of times in 1000 trials that various estimators of ρ beat the ITERPW estimator of ρ
A.8	Number of times in 1000 trials that estimated ρ equals boundary value
A.9	Number of iterations and failures to converge

Table A.1

EXACT THEORETICAL EFFICIENCY, RELATIVE TO OLS, OF ESTIMATORS THAT USE TRUE ρ

Independent Variable	Estimator	ρ = -.8 b_1	-.8 b_2	.0 b_1	.0 b_2	.4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
T=20													
t	TRUECO	1.17	1.20	.90	.93	.81	.86	.50	.62	.29	.42	.04	.11
	AITKEN	1.18	1.21	1.00	1.00	1.02	1.02	1.08	1.09	1.08	1.10	1.03	1.08
GNP_t	TRUECO	1.23	1.24	.93	.95	.88	.91	.71	.81	.57	.75	.29	.71
	AITKEN	1.24	1.24	1.00	1.00	1.02	1.02	1.13	1.14	1.16	1.20	1.13	1.26
CAP_t	TRUECO	1.75	1.74	.98	.97	1.10	1.10	1.85	1.83	2.10	2.19	1.04	2.51
	AITKEN	1.75	1.75	1.00	1.00	1.14	1.14	1.85	1.86	2.15	2.21	1.95	2.52
T=50													
GNP_t	TRUECO	1.14	1.14	.95	.95	.90	.91	.84	.87	.87	.94	.92	1.39
	AITKEN	1.14	1.14	1.00	1.00	1.02	1.02	1.19	1.19	1.40	1.41	1.78	1.89

Table A.2

EXPERIMENTAL EFFICIENCY OF VARIOUS ESTIMATORS RELATIVE TO OLS

Independent Variable	Estimator	ρ = -.8 b_1	b_2	.0 b_1	b_2	.4 b_1	b_2	.8 b_1	b_2	.9 b_1	b_2	.98 b_1	b_2
T=20													
t	TRUECO	1.17	1.21	.90	.92	.79	.84	.51	.63	.28	.41	.04	.11
	2SCO	1.16	1.19	.91	.92	.81	.86	.64	.77	.31	.62	.66	.74
	ITERCO	1.16	1.19	.91	.92	.80	.85	.51	.69	.27	.56	.54	.64
	AITKEN	1.18	1.21	1.00	1.00	1.02	1.02	1.09	1.09	1.07	1.11	1.05	1.07
	2SPW	1.18	1.21	.99	.99	1.01	1.01	1.05	1.06	1.05	1.08	1.02	1.05
	ITERPW	1.18	1.21	.99	.99	1.01	1.01	1.06	1.07	1.05	1.08	1.02	1.05
	BM	1.18	1.21	.99	.99	1.01	1.01	1.05	1.06	1.05	1.07	1.02	1.04
GNP_t	TRUECO	1.21	1.21	.94	.95	.90	.94	.72	.82	.58	.75	.28	.71
	2SCO	1.20	1.20	.94	.95	.91	.93	.84	.91	.87	.95	.95	1.03
	ITERCO	1.20	1.20	.94	.95	.73	.85	.59	.80	.51	.83	.52	.88
	AITKEN	1.22	1.21	1.00	1.00	1.02	1.03	1.12	1.14	1.17	1.21	1.15	1.28
	2SPW	1.21	1.21	.99	.99	1.01	1.01	1.08	1.09	1.10	1.13	1.06	1.12
	ITERPW	1.21	1.21	.99	.99	1.00	1.01	1.08	1.09	1.11	1.14	1.07	1.14
	BM	1.21	1.21	.99	.99	1.01	1.01	1.08	1.09	1.10	1.13	1.06	1.12
CAP_t	TRUECO	1.70	1.69	.98	.98	1.12	1.11	1.91	1.90	2.09	2.18	1.03	2.49
	2SCO	1.69	1.68	.97	.97	1.05	1.04	.01	1.41	.00	1.65	.00	1.83
	ITERCO	1.69	1.68	.95	.95	1.03	1.03	.01	1.65	.00	2.03	.00	2.27
	AITKEN	1.70	1.69	1.00	1.00	1.15	1.15	1.90	1.90	2.13	2.19	1.92	2.49
	2SPW	1.70	1.69	.99	.99	1.06	1.06	1.38	1.38	1.57	1.60	1.54	1.75
	ITERPW	1.70	1.69	.98	.98	1.04	1.05	1.61	1.61	1.94	2.00	1.77	2.15
	BM	1.70	1.69	.98	.98	1.05	1.06	1.58	1.58	1.92	1.97	1.75	2.12
T=50													
GNP_t	TRUECO	1.13	1.13	.96	.96	.91	.92	.83	.87	.83	.90	.90	1.33
	2SCO	1.13	1.13	.96	.96	.91	.92	.85	.88	.91	.95	.02	1.13
	ITERCO	1.13	1.13	.96	.96	.91	.92	.70	.77	.72	.84	.02	1.12
	AITKEN	1.14	1.15	1.00	1.00	1.02	1.02	1.21	1.20	1.38	1.39	1.75	1.84
	2SPW	1.14	1.15	1.00	1.00	1.02	1.02	1.15	1.14	1.28	1.28	1.41	1.44
	ITERPW	1.14	1.15	1.00	1.00	1.02	1.02	1.12	1.11	1.26	1.26	1.52	1.57
	BM	1.14	1.15	1.00	1.00	1.02	1.02	1.14	1.13	1.28	1.29	1.48	1.52

Table A.3

NUMBER OF TIMES IN 1000 TRIALS THAT VARIOUS ESTIMATORS BEAT OLS

Independent Variable	Estimator	ρ = -.8 b_1	-.8 b_2	.0 b_1	.0 b_2	.4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
T=20													
t	TRUECO	567	586	420	424	371	394	236	316	151	222	24	74
	2SCO	557	585	426	438	382	413	325	378	324	349	352	397
	ITERCO	557	584	424	434	381	410	321	372	318	340	347	383
	AITKEN	581	604	--	--	526	513	562	574	562	593	573	574
	2SPW	583	603	508	510	538	519	575	592	580	631	575	591
	ITERPW	579	601	508	508	537	517	572	589	583	631	577	591
	BM	584	603	508	510	539	523	581	597	592	632	576	593
GNP_t	TRUECO	592	608	461	457	443	461	382	441	324	398	192	374
	2SCO	591	605	458	457	443	457	421	468	433	454	438	495
	ITERCO	591	605	458	456	441	453	405	451	395	423	395	450
	AITKEN	610	618	--	--	532	550	582	576	604	618	594	620
	2SPW	608	619	484	467	528	536	610	622	645	656	619	658
	ITERPW	607	618	484	467	525	535	606	619	644	648	619	654
	BM	612	620	484	467	527	537	611	630	646	659	619	660
CAP_t	TRUECO	691	695	468	470	568	563	726	731	728	713	523	781
	2SCO	693	695	491	488	547	534	737	737	742	741	721	793
	ITERCO	692	694	487	484	532	521	722	725	720	732	703	786
	AITKEN	693	696	--	--	582	589	728	741	709	725	725	779
	2SPW	699	699	499	500	568	571	743	748	766	764	734	799
	ITERPW	698	698	492	496	559	561	722	737	750	748	726	784
	BM	700	701	496	496	561	562	736	741	755	757	731	790
T=50													
GNP_t	TRUECO	572	562	455	450	427	428	406	430	433	461	445	599
	2SCO	567	564	450	443	425	432	401	416	449	477	509	553
	ITERCO	567	564	450	443	425	430	387	400	399	429	423	512
	AITKEN	579	572	--	--	543	553	619	620	645	645	692	710
	2SPW	582	571	486	484	537	545	628	634	688	685	714	737
	ITERPW	580	570	486	484	535	545	620	628	674	672	708	728
	BM	583	573	486	484	537	545	630	637	685	689	721	741

Note: Counts greater than 531 or smaller than 469 are significantly different from 500 at the .05 level.

Table A.4

NUMBER OF TIMES IN 1000 TRIALS THAT VARIOUS ESTIMATORS BEAT ITERPW

Independent Variable	Estimator	ρ = -.8 b_1	-.8 b_2	.0 b_1	.0 b_2	.4 b_1	.4 b_2	.8 b_1	.8 b_2	.9 b_1	.9 b_2	.98 b_1	.98 b_2
T=20													
t	OLS	421	399	492	492	463	483	428	411	417	369	423	409
	TRUECO	470	482	412	430	387	400	242	299	149	218	24	75
	2SCO	471	488	422	431	402	414	343	363	325	319	357	381
	ITERCO	476	499	420	430	402	413	333	357	316	313	349	367
	AITKEN	503	506	492	492	517	503	534	537	517	546	563	542
	2SPW	521	509	509	512	493	514	449	445	430	420	458	439
	BM	515	500	504	505	489	510	446	445	450	406	433	432
GNP_t	OLS	393	382	516	533	475	465	394	381	356	352	381	346
	TRUECO	492	481	465	465	448	454	349	401	302	357	181	342
	2SCO	500	497	457	454	435	449	375	407	364	379	416	411
	ITERCO	499	495	455	453	433	444	362	392	347	370	376	382
	AITKEN	506	496	516	533	532	532	527	527	564	591	568	550
	2SPW	520	531	538	552	502	499	433	428	394	383	419	377
	BM	511	525	529	545	499	494	438	433	382	380	409	364
CAP_t	OLS	302	302	508	504	441	439	278	263	250	252	274	216
	TRUECO	526	523	510	509	519	514	552	547	517	519	310	521
	2SCO	527	520	485	491	473	464	456	464	412	427	401	436
	ITERCO	525	512	469	474	458	450	528	515	456	498	435	514
	AITKEN	500	496	508	504	561	565	561	564	523	531	539	538
	2SPW	503	501	527	531	500	500	398	390	399	423	434	417
	BM	490	486	526	531	512	514	430	419	436	449	427	423
T=50													
GNP_t	OLS	420	430	514	516	465	455	380	372	326	328	292	272
	TRUECO	472	474	449	451	442	442	361	380	354	365	344	435
	2SCO	466	469	454	451	427	431	356	368	343	354	319	345
	ITERCO	467	470	454	451	425	430	347	355	315	319	299	329
	AITKEN	474	480	514	516	490	494	564	556	553	561	586	597
	2SPW	519	526	549	550	500	491	440	432	426	415	383	354
	BM	516	522	517	519	493	484	441	445	427	415	384	353

Note: Counts greater than 531 or smaller than 469 are significantly different from 500 at the .05 level.

Table A.5

NUMBER OF TYPE 1 ERRORS IN 1000 TRIALS AT .05 SIGNIFICANCE LEVEL

Independent Variable	Estimator	ρ											
		-.8		.0		.4		.8		.9		.98	
		b_1	b_2	b_1	b_2	b_1	b_2	b_1	b_2	b_1	b_2	b_1	b_2
T=20													
t	OLS	0	0	42	44	193	197	502	490	645	571	848	709
	TRUECO	44	52	47	45	57	57	62	54	55	51	56	54
	2SCO	42	57	81	74	131	139	331	322	430	397	682	509
	ITERCO	43	61	81	73	131	138	322	318	421	390	670	503
	AITKEN	41	51	42	44	47	65	53	41	43	46	53	64
	2SPW	43	62	78	72	125	132	302	293	411	340	690	473
	ITERPW	49	65	78	72	124	131	293	285	401	336	700	474
	BM	41	55	78	71	126	133	312	305	433	360	731	503
GNP_t	OLS	0	0	63	59	186	185	457	449	601	596	730	666
	TRUECO	50	53	49	56	47	40	45	51	55	47	57	49
	2SCO	47	56	86	86	131	128	278	269	394	371	510	424
	ITERCO	48	57	86	87	127	129	281	271	392	378	519	426
	AITKEN	49	56	63	59	47	41	51	53	46	35	55	52
	2SPW	51	60	88	89	138	138	258	251	354	343	509	413
	ITERPW	52	64	91	89	136	136	254	246	343	322	486	397
	BM	48	56	88	88	139	138	261	258	375	352	534	432
CAP_t	OLS	3	3	52	51	147	143	304	294	341	323	407	322
	TRUECO	53	55	55	52	43	45	50	49	42	43	50	51
	2SCO	53	52	75	70	101	96	147	143	156	137	241	129
	ITERCO	54	55	78	75	107	100	93	96	94	78	198	85
	AITKEN	47	50	52	51	43	46	51	48	41	49	53	51
	2SPW	56	58	74	74	110	106	153	149	154	144	211	137
	ITERPW	60	59	80	78	115	113	102	101	90	86	144	86
	BM	54	56	78	74	112	109	107	107	98	92	176	86
T=50													
GNP_t	OLS	0	0	49	46	209	209	501	505	636	631	754	781
	TRUECO	50	50	48	45	55	54	49	44	57	56	50	59
	2SCO	48	52	67	72	94	99	163	164	241	246	397	375
	ITERCO	48	52	66	72	94	99	176	177	253	254	376	350
	AITKEN	52	48	49	46	51	45	40	45	52	55	54	59
	2SPW	51	52	69	74	90	92	143	143	198	202	377	366
	ITERPW	52	53	69	74	87	92	138	137	184	191	307	296
	BM	50	51	69	73	90	92	151	151	200	208	357	338

Note: Counts greater than 63 or smaller than 37 are significantly different from 50 at the .05 level.

Table A.6

PERFORMANCE OF ESTIMATORS OF ρ

Independent Variable	Estimator	ρ = -.8 M	-.8 RMSE	.0 M	.0 RMSE	.4 M	.4 RMSE	.8 M	.8 RMSE	.9 M	.9 RMSE	.98 M	.98 RMSE
T=20													
t	2SCO	-.75	.16	-.09	.23	.23	.28	.47	.40	.53	.43	.53	.49
	ITERCO	-.76	.16	-.09	.23	.23	.28	.48	.39	.54	.43	.55	.48
	2SPW	-.79	.15	-.09	.25	.24	.28	.50	.38	.57	.41	.59	.46
	ITERPW	-.81	.16	-.09	.25	.24	.29	.51	.37	.58	.40	.59	.45
	BM	-.76	.15	-.09	.24	.23	.28	.48	.39	.55	.42	.56	.47
GNP_t	2SCO	-.74	.17	-.10	.23	.20	.29	.47	.39	.52	.43	.55	.47
	ITERCO	-.75	.16	-.10	.24	.20	.30	.49	.38	.55	.42	.58	.46
	2SPW	-.78	.16	-.11	.25	.21	.30	.51	.37	.58	.40	.61	.44
	ITERPW	-.80	.16	-.11	.25	.22	.30	.52	.36	.59	.39	.63	.42
	BM	-.75	.16	-.10	.24	.21	.30	.49	.38	.56	.41	.59	.45
CAP_t	2SCO	-.73	.17	-.09	.23	.22	.28	.51	.38	.58	.40	.64	.42
	ITERCO	-.76	.16	-.08	.25	.25	.28	.59	.32	.67	.32	.73	.33
	2SPW	-.78	.16	-.09	.25	.23	.29	.54	.36	.62	.37	.68	.39
	ITERPW	-.80	.16	-.09	.26	.26	.29	.63	.31	.72	.30	.79	.30
	BM	-.76	.16	-.09	.25	.24	.29	.59	.32	.68	.32	.74	.33
T=50													
GNP_t	2SCO	-.78	.09	-.05	.14	.33	.15	.68	.17	.76	.18	.80	.20
	ITERCO	-.78	.09	-.05	.15	.33	.15	.69	.17	.77	.17	.83	.19
	2SPW	-.79	.09	-.05	.15	.34	.15	.70	.16	.79	.16	.84	.18
	ITERPW	-.80	.09	-.05	.15	.34	.15	.70	.15	.80	.15	.85	.17
	BM	-.78	.09	-.05	.15	.33	.15	.69	.16	.78	.16	.83	.18

Table A.7

NUMBER OF TIMES IN 1000 TRIALS THAT VARIOUS ESTIMATORS OF ρ BEAT THE ITERPW ESTIMATOR OF ρ

Independent Variable	Estimator	ρ = −.8	.0	.4	.8	.9	.98
T=20							
t	2SCO	545	1000	375	83	35	34
	ITERCO	557	821	470	236	201	226
	2SPW	588	1000	389	83	76	147
	BM	557	1000	381	92	39	14
GNP_t	2SCO	517	1000	372	76	35	18
	ITERCO	530	808	432	246	230	271
	2SPW	566	1000	381	96	101	114
	BM	531	1000	382	97	52	13
CAP_t	2SCO	515	1000	427	174	108	77
	ITERCO	564	728	545	372	268	236
	2SPW	568	999	437	200	131	174
	BM	566	1000	454	276	195	64
T=50							
GNP_t	2SCO	511	1000	342	180	107	37
	ITERCO	516	760	382	318	287	298
	2SPW	541	1000	351	219	194	198
	BM	512	1000	349	197	133	39

Note: Counts greater than 531 or smaller than 469 are significantly different from 500 at the .05 level.

Table A.8

NUMBER OF TIMES IN 1000 TRIALS THAT ESTIMATED ρ EQUALS BOUNDARY VALUE

Independent Variable	Estimator	ρ: -.8	.0	.4	.8	.9	.98
T=20							
t	2SCO	3	0	0	0	0	0
	ITERCO	11	0	0	0	0	0
	2SPW	14	0	0	2	5	13
	ITERPW	58	0	0	0	0	0
	BM	0	0	0	0	0	0
GNP_t	2SCO	0	0	0	0	0	0
	ITERCO	7	0	0	0	0	0
	2SPW	12	0	0	3	8	14
	ITERPW	53	0	0	0	1	6
	BM	0	0	0	0	0	0
CAP_t	2SCO	4	0	0	3	10	35
	ITERCO	8	0	0	3	10	35
	2SPW	27	0	0	8	29	95
	ITERPW	51	0	0	10	43	136
	BM	0	0	0	0	0	0
T=50							
GNP_t	2SCO	0	0	0	0	0	3
	ITERCO	0	0	0	0	0	0
	2SPW	1	0	0	0	2	16
	ITERPW	1	0	0	0	7	37
	BM	0	0	0	0	0	0

Table A.9

NUMBER OF ITERATIONS AND FAILURES TO CONVERGE

		ρ																	
		-.8			.0			.4			.8			.9			.98		
ependent riable	Estimator	M	SD	NC	M	SD	NC	M	SD	NC	M	SD	NC	M	SD	NC	M	SD	NC
$\underline{O}$	ITERCO	3.7	.6	0	3.9	.9	0	4.6	1.9	0	7.2	7.4	14	9.4	10.6	38	10.7	12.4	60
	ITERPW	3.8	.7	0	3.8	.8	0	4.3	1.2	0	5.1	1.6	0	5.3	1.6	0	5.5	1.7	0
	BM	3.8	.6	0	3.8	.8	0	4.2	1.0	0	4.9	1.4	0	5.0	1.3	0	5.2	1.4	0
NP_t	ITERCO	3.7	.6	0	3.9	.8	0	4.6	3.7	4	7.7	8.8	23	10.0	11.9	54	11.8	13.7	83
	ITERPW	3.8	.7	0	3.9	.9	0	4.4	1.5	0	5.4	2.2	0	6.1	2.7	0	6.7	3.7	0
	BM	3.8	.6	0	3.8	.8	0	4.2	1.2	0	5.0	1.7	0	5.5	1.9	0	5.9	2.2	0
AP_t	ITERCO	3.9	.6	0	5.0	2.9	0	6.4	4.6	2	8.5	6.7	7	10.6	10.2	39	13.0	12.8	66
	ITERPW	3.7	.6	0	4.8	2.4	0	6.2	4.1	2	7.0	4.4	2	6.7	3.4	0	6.2	4.0	1
	BM	3.8	.5	0	4.7	2.3	0	6.0	3.5	0	6.9	4.2	1	6.7	3.6	1	6.4	3.7	0
$\frac{O}{NP_t}$	ITERCO	3.1	.4	0	3.2	.4	0	3.5	.7	0	6.0	6.2	7	7.9	8.1	16	11.8	12.8	63
	ITERPW	3.2	.4	0	3.1	.4	0	3.5	.7	0	4.8	2.3	0	5.9	3.8	1	7.4	5.1	2
	BM	3.2	.4	0	3.1	.4	0	3.5	.7	0	4.6	1.7	0	5.4	3.1	1	6.8	4.2	1

Notes: M is the mean number of iterations. SD is the standard deviation of the number of iterations. NC is the number of times in 1000 trials that the iteration process failed to converge before reaching the iteration limit (52).

REFERENCES

Beach, Charles M., and James G. MacKinnon, "A Maximum Likelihood Procedure for Regression with Autocorrelated Errors," *Econometrica*, Vol. 46, No. 1, January 1978.

Cochrane, D., and G. H. Orcutt, "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," *American Statistical Association Journal*, Vol. 44, No. 245, March 1949.

Durbin, J., "The Fitting of Time Series Models," *International Statistical Institute Review*, Vol. 28, No. 3, 1960.

Harvey, A. C., and I. D. McAvinchey, *The Small Sample Efficiency of Two-Step Estimators in Regression Models with Autoregressive Disturbances*, Discussion Paper No. 78-10, University of British Columbia, April 1978.

Maeshiro, Asatoshi, "Autogressive Transformations, Trended Independent Variables and Autocorrelated Disturbance Terms," *Review of Economics and Statistics*, Vol. 58, No. 4, November 1976.

_______, "On the Retention of the First Observations in Serial Correlation Adjustment of Regression Models," *International Economic Review*, Vol. 20, No. 1, February 1979.

Park, R. E., and B. M. Mitchell, *Estimating the Autocorrelated Error Model with Trended Data*, R-2273-NIE/RC, The Rand Corporation, March 1978.

_______, *Maximum Likelihood vs. Minimum Sum-of-Squares Estimation of the Autocorrelated Error Model*, N-1325, The Rand Corporation, November 1979.

Prais, G. J., and C. B. Winsten, *Trend Estimators and Serial Correlation*, Discussion Paper No. 383, Cowles Commission, Chicago, 1954.

Rao, Potluri, and Zvi Griliches, "Small-Sample Properties of Several Two-Stage Regression Methods in the Context of Auto-Correlated Errors," *American Statistical Association Journal*, Vol. 64, No. 325, March 1969.

Spitzer, John J., "Small-Sample Properties of Nonlinear Least Squares and Maximum Likelihood Estimators in the Context of Autocorrelated Errors," *Journal of the American Statistical Association*, Vol. 74, No. 365, March 1979.

Theil, Henri, *Principles of Econometrics*, Wiley, New York, 1971.

DATE
FILMED

3-8